

# Chebyshev Functional Link Artificial Neural Networks for Denoising of Image Corrupted by Salt and Pepper Noise

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**Abstract**— Here we have presented an alternate ANN structure called functional link ANN (FLANN) for image denoising. In contrast to a feed forward ANN structure i.e. a multilayer perceptron (MLP), the FLANN is basically a single layer structure in which non-linearity is introduced by enhancing the input pattern with nonlinear function expansion. In this work three different expansions are applied. With the proper choice of functional expansion in a FLANN, this network performs as good as and in some case even better than the MLP structure for the problem of denoising of an image corrupted with Salt and Pepper noise. In the single layer functional link ANN (FLANN) the need of hidden layer is eliminated. The novelty of this structure is that it requires much less computation than that of MLP. In the presence of additive white Gaussian noise in the image, the performance of the proposed network is found superior to that of a MLP. In particular FLANN structure with Chebyshev functional expansion works best for Salt and Pepper noise suppression from an image.

**Index Terms**—MLP, FLANN, Chebyshev FLANN, Salt and Pepper noise.

## I. INTRODUCTION

DENOISING of image is a major field of image processing. When data is transmitted in channel, noise gets added in the image, it varies from time to time and also it changes in a fraction of second. A human expert can't take decision to choose a filter to suppress the noise at that small time. To avoid different limitations of fixed filters, adaptive filters are designed that adapt themselves to the changing conditions of signal and noise. In such an application, the image filter must adapt the image local statistics, the noise type, and the noise power level and it must adjust itself to change its characteristics so that the overall filtering performance has been enhanced to a high level. One of the most important example of it is neural network based adaptive image filter.

Artificial neural networks (ANN) have emerged as a powerful learning technique to perform complex tasks in highly nonlinear environment [1]. Some of the

advantage of ANN model are : (i) There ability to learn based on optimization technique of an appropriate error function, (ii) There excellent performance for approximation of nonlinear functions. Most of the ANN based systems are based on multilayer feed forward networks such as MLP trained with back propagation (BP). This is due to the fact that these networks are robust and effective in denoising of image. As an alternative to the MLP, there has been considerable interest in radial basis function (RBF) network in [2].

The functional link artificial neural network (FLANN) by pao [5] can be used for function approximation and pattern classification with faster convergence and lesser computational complexity than a MLP network. A FLANN using sine and cosine functions for functional expansion for the problem of nonlinear dynamic system identification has been reported [6]. For functional expansion of the input pattern, we choose the trigonometric, exponential, Chebyshev expansion and compare the outputs with MLP. The primary purpose of this paper is to highlight the effectiveness of the proposed simple ANN structure in the problem of denoising of image corrupted with Salt and Pepper noise.

## II. STRUCTURE OF THE ARTIFICIAL NEURAL NETWORK FILTERS

Here, we briefly describe the architecture and learning algorithm for multilayer neural network and FLANN.

### A. Multilayer perceptron

The MLP has a multilayer architecture with one or more hidden layers between its input and output layers. All the nodes of a lower layer are connected with all the nodes of the adjacent layer through a set of weights. All the nodes in all layers (except the input layer) of the MLP contain a nonlinear  $\tanh(\cdot)$  function. A pattern is applied to the input layer, but no computation takes place in this layer. Thus the output of the nodes of this layer is the input pattern itself. The weighted sum of outputs of a lower layer is passed through the nonlinear function of a node in the upper layer to produce its

output. Thus, the outputs of all the nodes of the network are computed. The outputs of the output layer are compared with a target pattern associated with the input pattern. The error between the target pattern and the output layer node is used to update the weights of the network. The MSE is used as a cost function and BP algorithm attempts to minimize the cost function by updating all weights of the network [1].

### B. Functional link ANN

The FLANN, which is initially proposed by Pao, is a single layer artificial neural network structure capable of performing complex decision regions by generating nonlinear decision boundaries. In a FLANN the need of hidden layer is removed. In contrast to linear weighting of the input pattern produced by the linear links of a MLP, the functional link acts on the entire pattern by generating a set of linearly independent functions. If network has two input i.e.

$$X = [x_1 \ x_2]^T$$

An enhanced pattern obtained by using functional expansion is given by

$$X = [1 \ x_1 \ T_1(x) \ x_2 \ T_2(x_2) \ \dots]^T. \quad (1)$$

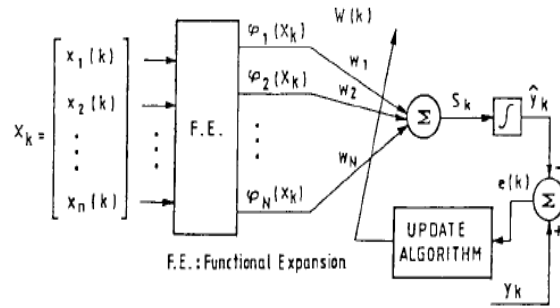


Figure 1. A FLANN structure.

In this paper the input pattern of the noisy image is sent in the input node of the FLANN structure and an enhanced pattern is obtained. The target will be the corresponding single pixel from original image. This process continues iteratively till all pattern of the image gets completed. The whole process continues for 100 times to find out error power with iteration. The BP algorithm used to train the FLANN becomes simple and has a faster convergence due to its single layer architecture. For functional expansion of the input pattern, the trigonometric, power series, exponential polynomials are chosen individually.

### C. Different functional expansions

Here the functional expansion block make use of a functional model comprising of a subset of orthogonal

sine and cosine basic functions and the original pattern along with its outer products. For example, considering a two dimensional input pattern i.e.

$$X = [x_1 \ x_2]^T$$

the enhanced pattern is obtained by using a trigonometric functions as

$$X^1 = [x_1 \cos(\Gamma x_1) \sin(\Gamma x_1) \dots x_2 \cos(\Gamma x_2) \sin(\Gamma x_2) \dots x_1 x_2]^T \quad (2)$$

Using exponential expansion will be

$$X^1 = [x_1 \exp^{x_1} \exp^{x_1^2} \dots x_2 \exp^{x_2} \exp^{x_2^2} \dots]^T \quad (3)$$

The Chebyshev polynomials are a set of orthogonal polynomials defined as the solution to the Chebyshev differential equation. The structure of a ChNN is shown in Fig. 2.

These higher Chebyshev polynomials for  $-1 < x < 1$  may be generated using the recursive formula given by

$$T_{n+1} = 2xT_n(x) - T_{n-1}(x) \quad (4)$$

The first few Chebyshev polynomials are given by

$$\left. \begin{aligned} T_0(x) &= 1 \\ T_1(x) &= x \\ T_2(x) &= 2x^2 - 1 \\ T_3(x) &= 4x^3 - 3x \\ T_4(x) &= 8x^4 - 8x^2 + 1 \\ T_5(x) &= 16x^5 - 20x^3 + 5x \end{aligned} \right\} \quad (5)$$

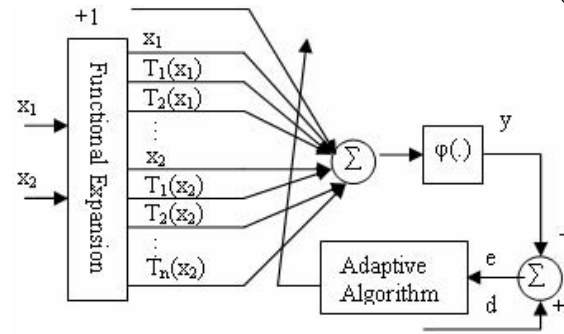


Figure 2. A ChNN structure.

Exponential polynomial expansion needs less number of computations and is very easy to implement than other three type of polynomial expansion. Chebyshev polynomial expansion gives better performance for the prediction of financial time series.

### III. COMPUTATIONAL COMPLEXITY

Here, we present a comparison of the computational complexity between an MLP, and a FLANN having different expansions and all having the  $\tanh(\cdot)$  as their nonlinear function. In all the cases, multiplications, additions and computations of the  $\tanh(\cdot)$  are required. However in the case of FLANN, additional computations of the sine and cosine functions are needed for its functional expansion. In the training and updating of the weights of MLP, extra computations are incurred due to its hidden layer. This is due to the error propagation for the calculation of the square error derivative of each neuron in the hidden layer. For each iteration the computation are: (1) Forward calculations to find the activation value of all the nodes of the entire network.(2) Back-error propagation for calculation of square error derivatives.(3) Updating weights of the entire network. In the case of MLP with  $\{I-J-K\}$ , the total number of weights is given by  $(I+1)J + (J+1)K$ . Whereas, in the case of FLANN with  $\{D-K\}$ , it is given by  $(D+1)K$ . The number of computation for both MLP and FLANN are shown in Table 1:

TABLE 1  
COMPARISON OF COMPUTATIONAL COMPLEXITY IN ONE ITERATION

Operation	MLP {I-J-K}	FLANN {D-K}
Addition	$2IJ + 3JK + 3K$	$2K(D+1) + K$
Multiplication	$3IJ + 4JK + 3J + 5K$	$3K(D+1) + 2K$
Tanh( $\cdot$ )	$J+K$	$K$

From this table it may be seen that the number of additions, multiplications and computation of tanh are much less in case of a FLANN than that of a MLP network. As the number of hidden layer increases the computations in a MLP increases. But due to absence of hidden layer in the FLANN its computational complexity reduces drastically.

### IV. SIMULATION STUDIES

Extensive simulation studies were carried out with several examples to compare performance of MLP with FLANN for denoising of image. This work is carried out when the types of noise is Salt and Pepper noise. In our simulation we set MLP to be  $\{9-4-1\}$ . Different parameters are decided after experimenting with different values of the parameters. It is observed that large window size, more hidden layer or more number of hidden layer neuron does not sure to produce better results. In all types of FLANN the input pattern is expanded in such a way that the total numbers of weights in the three ANNs are approximately same. The structure of FLANN is  $\{9-1\}$  and ease input of the input pattern was expanded five times using different expansion. Hence the total number of weights for the MLP and FLANN having different expansion will be

same and be equal to 45. The learning rate for ANN and FLANN is set at 0.03. The number of iteration was set to 3000 for all the models. The BP learning algorithm has been used. MATLAB simulation tool has been implemented here. The training inputs and corresponding targets were normalized to fall within the interval of  $[0, 1]$ . The MLP has logistic sigmoid nonlinear function at the hidden layer. In all the cases the output node has tan hyperbolic nonlinear function. For the training the neural network, we uses the back propagation algorithm. It is supervised learning, hence test image to which additive noise has been applied have been used. While training, the noisy pixels of  $3 \times 3$  window form the noisy image will be entered into the network as a vector. The associated desire value is the corresponding pixel value from original image. For this the network do not take into account the border values of the noisy image. Here the images taken are  $256 \times 256$  size and hence the network input vector is of  $253 \times 253$  image. For the training of network, a different intensity combination that may arise from noisy image is used. For this Lena image is used which is rich in different patterns. It is important to note that the neural network has a general training and can be applied to any kind of image with Salt and Pepper noise. Hence the network trained with any noisy image and can be tested with any noisy image.

#### A. Peak Signal to Noise Ratio

TABLE 2  
RESULT FOR FILTERS IN TERMS OF PSNR VALUE

	Noisy	ANN	Ch-NN
Image 1	20.28	26.52	27.42
Image 2	20.54	26.77	27.42
Image 3	20.43	26.67	27.36
Image 4	20.74	26.12	27.28

In this work computer simulations are carried out to compare the PSNR value of filtered images obtained from these adaptive models. Images were corrupted by Salt and Pepper noise of density 0.05, before filtration

The shown numbers corresponds to the peak signal-to- noise ratio PSNR value of Images. From this table it can be seen, the non linear adaptive filter FLANN having Chebyshev functional expansion have shows better result then MLP or any other expansion in FLANN in all the images. Table shows the result obtained when applying the neural network to a set of standard testing images. These images are shown in the figure.



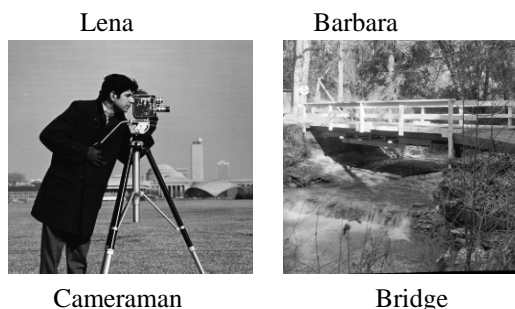


Figure 2. Original Images

### B. The Convergence Characteristics

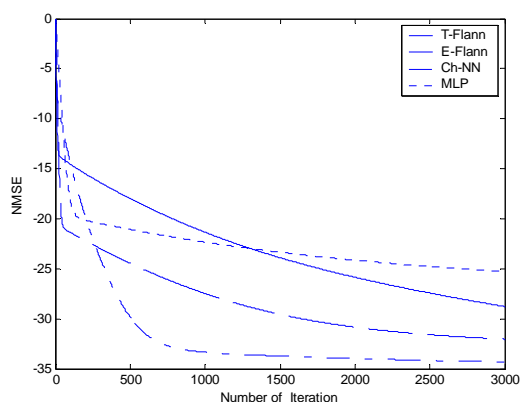


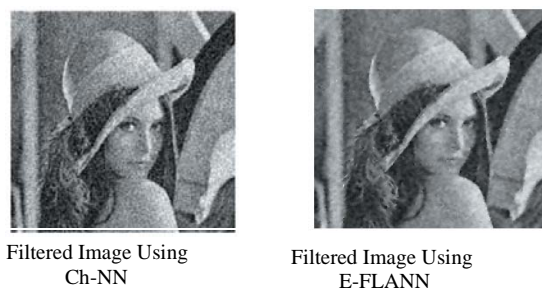
Figure 5. Convergence Characteristics of ANN and Three Different FLANN

The general convergence characteristics of ANN and FLANN having different expansion are shown in the figure 5.

T-FLANN: FLANN having Trigonometric expansion.  
E-FLANN: FLANN having Exponential expansion.  
P-Ch-NN: FLANN having Chebyshev expansion. The convergence characteristics for ANN and FLANN having different expansion are depicted here. It can be observed that FLANN having Chebyshev expansion shows much better convergence rate and lower MSE floor than other FLANN and ANN. It shows its superior performance in terms of convergence speed and steady state MSE level.

### C. Subjective Evaluation

The performance of ANN and FLANN structure with different expansion can also be judge by subjective evaluation i.e. from seeing the noise free image.



Filtered Image Using Ch-NN

Filtered Image Using E-FLANN



Figure 4. Filtered Images

Filtered Image Using T-FLANN

Filtered Image Using MLP

### D. The Computational Complexity.

The computational complexity of ANN with FLANN are analyzed and compared in the table. It can be seen that the number of additions are almost same in ANN and FLANN structure but the number of multiplication and computation of  $\tanh(\cdot)$  function is much lesser than that of the MLP.

TABLE.3.

COMPARISON OF COMPUTATIONAL COMPLEXITY IN ONE ITERATION

Number of Operation	MLP 9-4-1	FLANN 45-1
Addition	$2 \times 9 \times 4 + 3 \times 4 \times 1 + 3 \times 1 = 87$	$2 \times 1(45+1) + 1 = 93$
Multiplication	$3 \times 9 \times 4 + 4 \times 4 \times 1 + 3 \times 4 + 5 \times 1 = 141$	$3 \times 1(45+1) + 2 \times 1 = 140$
Tanh(.)	$4 + 1 = 5$	1

### E. CPU Training Time.

The training time is the average time taken for the completion of the training phase of each of the ANNs on a computer with the specification of AMD 1.8 GHz processor and 1024 MB of RAM.

TABLE.4.  
COMPARISON OF TRAINING TIME BETWEEN THE ANN AND C-FLANN

Avg, Training Time(s)	MLP {9-4-1}	C-FLANN {9-1}
3000 iteration	454.5	211.64
1000 iteration	152.83	71.17

From the table it is shown that the MLP requires about 152 second for training with 1000 iteration. But in FLANN it needs about 70 second. The average time require to compute the expansions of polynomials were found to be about 4 second.

## V. CONCLUSION

Here we have proposed use of single layer FLANN structure which is computationally efficient for denoising of image corrupted with Salt and Pepper noise. The functional expansion may be thought of analogous to the nonlinear processing of signals in the hidden layer of an MLP. This functional expansion of the input increases the dimension of the input pattern. In the FLANN structure proposed for denoising of image, the input functional expansion is carried out using the trigonometric, exponential or Chebyshev polynomials. The prime advantage of the FLANN structure is that it reduces the computational complexity without any sacrifice on its performance.

Simulation results indicate that the performance of FLANN is better than MLP for Salt and Pepper noise suppression from an image. From these work it is clear that FLANN having Chebyshev Functional expansion is better for Salt and Pepper noise suppression than other FLANN structure. The FLANN structure having Chebyshev functional expansion may be used for online image processing application due to its less computational requirement and satisfactory performance. The new nonlinear adaptive filter FLANN shown satisfactory results in its application to images with additive noise. Its adaptive capacity to different parameters when generating the image with Gaussian noise has to be studied. Generalization of this filter applicable to other types of noise has to be developed.

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